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Does voluntary participation undermine the Coase Theorem?

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Abstract

The Coase Theorem states that costless enforcement of voluntary agreements yields efficient outcomes. We argue that previous treatments fail to recognize the full meaning of 'voluntary'. It requires a two-stage game: a non-cooperative participation decision, followed by Coaseian bargaining only among those who choose to participate. We illustrate this in a simple public-goods model, and find outcomes ranging from extremely inefficient to fully efficient. However, the efficient equilibrium is not robust to even very small transaction costs. Thus, we cast doubt on Coaseian claims of universal efficiency. Finally, we outline a kind of coercion that restores efficiency. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

In his article 'The Problem of Social Cost,' Ronald Coase introduced a very powerful idea of great importance. Coase's article has been arguably the single

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†Footnote by Avinash Dixit: Mancur Olson was Distinguished Professor of Economics and Director of the Center for Institutional Reform and the Informal Sector (IRIS) at the University of Maryland. He died very suddenly on February 19, 1998, in the midst of numerous projects including a revision of this paper. I have had to prepare the final version without the benefit of his insight, scholarship, energy, and enthusiasm.

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largest influence on thinking about economic policy for the last three decades. It is one of the most — if not the most — widely cited economics article in recent times.[1]

Coase argued that, given a precise allocation of property rights and the absence of any costs of information or negotiation, two parties would arrive at a bargain that would internalize any externalities between them. Though Coase took for granted a government that allocated the property rights between the parties and a court that enforced their agreed bargain, he emphasized that an efficient outcome would occur whatever the initial allocation of legal rights.[2] Coase extended his analysis beyond two-party externalities to larger groups and even to 'amorphous' externalities or public bads like air pollution (see the Coase, 1988 book, in which the Coase, 1960 article is reprinted, pp. 24–25, 170–177, 180–182). While it is admitted that transaction costs will increase when the number of people in the group impacted by the externalities or served by the public good are large, the argument is that in Coase's idealized world of zero transaction costs, efficient outcomes can be achieved no matter how large the numbers. Thus, Coase's argument applies to public goods for large numbers as well as to local externalities. In addition, using his own earlier theory of the firm, Coase argued in Section VI of his article (Coase, 1960) that economic activity will be carried out by whatever means, market or non-market, that minimizes total costs: that is, production plus transaction costs. In short, the Pigouvian argument that government is needed to use taxes and subsidies to internalize externalities was fundamentally unsatisfactory; even in the presence of externalities and public goods, the rational bargaining of the parties in the economy would bring efficiency without any governmental intervention.

1.1. The narrow and the broad theorem

Coase did not claim he had offered a theorem, but George Stigler and legions of other economists have asserted that he had. Therefore, they attribute to him a deductive result that is, within its domain of application, necessarily and universally true. Though in some formulations of the theorem there are also other claims, the most basic claim of what has come to be called the Coase Theorem is that only transaction (or bargaining) costs can prevent voluntary bargaining from attaining Pareto-efficient outcomes. The theorem can be fairly stated as follows: 'If

[1] According to the Social Science Citation Index volumes since 1972, even Milton Friedman and Paul Samuelson do not have a single publication that has been cited even half as often as 'The Problem of Social Cost.' This article was also, by a huge margin, the most widely ordered article in the Bobbs–Merrill Reprint Series in Economics.

[2] The initial distribution of legal rights affects the distribution of income, and, thus, the outcomes may be different because of income effects.

transaction costs are zero, rational parties will necessarily achieve a Pareto-efficient allocation through voluntary transactions or bargaining.'

Different economists define transaction costs differently, but all agree that the resources devoted to transactions have alternative uses and, thus, an opportunity cost. Therefore, transaction costs must be taken into account in defining the Pareto frontier. When this point is used along with a comprehensive definition of transaction costs, the Coase Theorem can easily be transformed into an even grander proposition. If the familiar Coase theorem is true, it must also be true that rational parties in an economy will make all those trades in private goods, and all those bargains to internalize externalities, provide public goods, and deal with any other potential market failures, that bring positive **net** gains — that is, gains greater than the transaction costs needed to realize them. They will not make those deals that cost more to make than they are worth, and obviously Pareto efficiency requires that such deals should not be made. Thus, if the Coase Theorem is true, so is a 'super Coase Theorem,' namely that 'rational parties will necessarily achieve a Pareto-efficient allocation through voluntary transactions or bargaining, no matter how high transaction costs might be.'

When transaction costs are important, so is the transaction technology. There are obvious incentives to come up with innovations that reduce transaction costs. The above argument then extends to say that the most cost-effective methods of reducing transaction costs will get chosen. Some innovations that reduce transaction costs are organizational rather than technological: for example money, which eliminates the transaction cost of barter which requires a double coincidence of wants. Then the theorem says that such institutions will emerge through the same process of voluntary transaction.

1.2. The theorem applied to politics

Some followers of Coase, for example Cheung (1970), have taken the next logical step and pointed out that government is an organization that can reduce transaction costs. Coase also recognizes the possibility that governments, though their policies in practice typically have serious defects, could sometimes in dealing with certain problems have lower transaction costs than the private sector (Coase, 1988, p. 27). Then the comprehensive Coase Theorem extends to cover politics: rational actors in the polity will bargain politically until all mutual gains have been realized. Therefore, democratic government produces socially efficient results. It is not even necessary to start by postulating the existence of such a government; if it does not exist, but its value to the society exceeds the transaction costs of setting it up and operating it, then it will emerge through Coaseian bargaining.

A number of economists, some of whom began with strong classical-liberal or conservative world views, have thus been led, with impressive scientific honesty, by their understanding of the logic of the Coase Theorem, to an astonishingly optimistic account of economic policy in democratic governments. Notable examples are Stigler (1971, 1992) and Wittman (1989, 1995).

1.3. Numbers matter

As Olson (1996) has argued, these Panglossian implications of the Coase Theorem are difficult to reconcile with the historical record. History is not only full of examples of egregiously wasteful economic policies, but also of destruction and violence, such as in holocausts and wars, that are certainly not Pareto-efficient and cannot be consistent with the Coase Theorem. Olson (1965), and several others following him, for example Hardin (1982) and Sandler (1992), have argued that the Coase Theorem often leads to absurd conclusions because it does not take account of the way that an increase in the number who must participate in the internalization of an externality or the provision of a public good makes it difficult or impossible for Coaseian bargaining to achieve Pareto efficiency. The point is not merely that transaction costs increase with numbers; that would be covered by the 'super-Coase' formulation. Rather, the key to the argument is the familiar economic problem of free riding.

The argument goes as follows. As the number who would benefit from provision of a non-exclusive public good increases, other things being equal, voluntary non-cooperative rational individual behavior leads any group to fall further short of obtaining a group-optimal level of provision. In a society consisting of N identical individuals, each gets only $(1/N)$th of the total benefit of the good; therefore, each contributes too little. This problem gets worse as N increases. If the individuals differ in their intensity of demand for the public good, those with the strongest demands will contribute more than their mere numbers would indicate, but the total will still fall short of the social optimum.

1.4. What is a voluntary agreement?

But a non-cooperative contribution equilibrium does not allow for Coaseian bargaining, and Coaseians claim that a meeting of all potential beneficiaries will achieve unanimous agreement for a fully efficient provision of the public good. We argue that in this they fail to recognize the deep basis of free riding. It is an inherent consequence of the Coaseian requirement that agreements be voluntary. Therefore, it arises in the very act of convening such a meeting.

Suppose that the benefits accruing to M people would suffice to cover the cost of providing the public good, where M is less than N, the total population. Then no individual is pivotal or critical to the outcome. Any one can reckon that if he stays away from the meeting, the remaining $(N-1)$ will find it worth their while to contribute and provide the good anyway. Then the absentee can enjoy the benefits of the good (remember it is non-excludable) without paying any of the cost (remember that agreement has to be voluntary, so someone who was absent and did not consent cannot be compelled to pay). The potential benefit of such free riding can tempt every member of the population. If there are enough such free riders, the bus will stay in the garage — the public good will not be provided even though its total social benefit may exceed its cost by a considerable margin.

Adherents of the Coase Theorem propose to get around this free rider problem by using more complex conditional agreements of the form 'Each person will be asked to pay his share if and only if all others pay their shares,' or 'If anyone is absent from the meeting, the good will not be provided at all.' If such resolutions were credible prior commitments, then an individual contemplating staying away would recognize that he is indeed pivotal — his absence would kill the project and he would not enjoy the free rider's benefit. But remember that it is the meeting that will decide the matter. At the time the individual is deciding whether to participate, the meeting has not yet taken place, and no commitment to an 'all-or-nothing' choice has been made. If an individual is to expect that the future meeting will make such a choice, it has to be ex post optimal for the meeting to do so. In other words, it has to be a part of a properly specified forward-looking or subgame-perfect equilibrium of a two-stage game, of which the first stage is the non-cooperative choice of isolated individuals as to whether to attend a meeting, and the second stage is the cooperative action of those who have turned up for the meeting.[3]

This point is basic to the voluntary nature of Coaseian bargains. Individuals should have the right to decide freely whether to participate in them. Once participants have emerged, and have struck a deal, it will be enforced by the prevailing transactions technology. But in the strict logic of the argument, there is no such thing as society until individuals come together to form it, and statements such as 'the society will devise a conditional contract to ensure efficiency' are empty until one specifies the decision process of individuals that leads to the formation of this society.

All previous Coaseian approaches to public good provision share this defect in one way or another: they assume that all potential beneficiaries of the good have already gathered together, and proceed to analyze whether and how they can arrive at efficient solutions, constrained by the transaction cost technology. The core — for example Foley (1970), Mas-Colell (1980), and Cornes and Sandler (1986, pp. 303–306, 417–419) — is an explicitly cooperative concept of this kind. Mechanism design approaches — for example Clarke (1971), Groves and Ledyard (1977), and Cornes and Sandler (1986, Chap. 7) — start by assuming that the power to make and implement the mechanism has been handed over to someone, presumably by a duly constituted meeting of all potential participants. They do not

[3]By making the meeting a first stage, with a clearly understood time and venue, we are actually biasing the situation in favor of an efficient Coaseian outcome. Without this assumption, the act of organizing a first stage is itself a problem of providing a public good, so we cannot begin to solve the main public good problem without solving this prior one!

consider individual incentives to attend this meeting; thus, they skip what we called the vital first stage of voluntary participation.

1.5. Foreshadowing the model

In Section 2 we develop a formal model that captures what we have argued as the essential and basic voluntary participation choice. Here we briefly discuss the structure of the model and the intuition for its results.

To examine the Coase Theorem in its simplest and purest form, we assume that there are no transaction costs whatever and that all agreements are costlessly and reliably enforced. We also simplify further by assuming that the public good is available in only one discrete quantity, so that we do not need to consider any bargaining about partial provision or about how much to obtain. Any obstacles to efficiency which we discover in this framework can only be magnified in more general settings with transaction costs and continuously variable amounts of the good.

We believe that it is essential to respect both aspects: voluntary participation, and costless enforcement. We do this using a two-stage game. The first stage is non-cooperative, where isolated individuals decide whether to participate in the second stage. The latter is the familiar Coaseian bargaining process, and can be modelled using any of the standard approaches like the core or mechanism design. Since we do not consider transaction costs, for us these are all equivalent, and produce an outcome that is optimal for those who have chosen to participate. But that in turn profoundly affects the Stage 1 participation decision, and, therefore, the efficiency of the outcome for society as a whole.

Consider a non-excludable, discrete public good that provides a benefit V to every person in some group of N individuals and costs C to produce. Suppose initially that all individuals are identical. Let M be the smallest integer such that $MV > C$; thus, M is the smallest group that would find it advantageous to produce the good entirely at its own expense. Often M will be smaller than N. This will be the case whenever the good provides enough of a surplus over its costs so that a subset of those who benefit from provision would gain from providing it even if they bore all of the costs. It will also be true whenever a public good that already is worth enough to cover its costs is non-rival (i.e. such that additional consumers do not reduce the consumption of others) and new people move into the group or community. When the gain to the M individuals exceeds the cost C of the good, it is a fortiori necessary for Pareto efficiency that it be provided when $N > M$.

Suppose some n individuals of size M or larger provide the good. They must still solve a bargaining problem among themselves to share the cost. Since there is no information asymmetry, we assume that they do so efficiently. For the most part, in fact, we assume that the individuals are also identical as regards their bargaining abilities, that is, they share the costs equally, and each individual pays C/n. However, in one crucial context this assumption contributes to a more optimistic conclusion about the good being provided. At that point we will return to the assumption and comment upon it.

Consider the N individuals, initially in isolation, making independent decisions about whether they will participate in the provision of a public good or even in any discussions about mechanisms or agreements that might provide the good. We can quickly get some intuitive sense of the matter by comparing the gains to an individual if he chooses to participate and those that come if he attempts to free ride on the provision of others.

First consider a single play of this two-stage game. As usual, the subgame perfect equilibrium is found by starting with Stage 2, namely the Coaseian bargaining or mechanism design among those who have chosen to participate. If M or more people show up at Stage 2, it is optimal for them to proceed to provide the good and share the cost. Thus, at least in this paper, we have no quarrel with the presumption of efficiency of Coaseian bargaining once it starts. The difficulty which we emphasize arises from free riding at the previous stage where individuals are deciding whether to participate in the whole process, rationally looking ahead to what will happen at Stage 2.

Focus on one person, whom we call Herb for sake of definiteness. The only gain to Herb from being a contributor arises in the eventuality where exactly $M-1$ others decide to contribute. In this case Herb's contribution is pivotal and his gain from contributing is $V-C/M$. The gain to Herb from deciding to free ride is that he obtains the larger gain of V if M or more of the others decide to contribute. When N exceeds M by much, the likelihood that Herb will be the pivotal contributor and, thus, gain from the decision to contribute is small; Herb is much more likely, if there is provision, to be a non-indispensable contributor, and in all such cases he is wiser to have made the decision to be a free rider. The model allows us to quantify these ideas, and shows, for example, that when $M=10$ and $N=30$, the likelihood that there will be provision is much less than one in a million.

However, limiting the game to a single play of the two stages is arbitrary, especially since we have assumed zero transaction costs. Therefore, we go on to consider a repeated version of the two-stage game. We find two equilibria, each sustained by its own internally consistent expectations. In the first equilibrium, individuals at every play of Stage 1 believe that if M or more people turn up at the immediately following Stage 2 they will go ahead and provide the good on their own. This further reduces the incentive for any one individual to attend the meeting at any one play: if an error is made (too few people show up) there will be opportunities to play again. Because of this reduction of individual participation, paradoxically, repetition actually makes the outcome even less efficient. In the second equilibrium, everyone at every play of Stage 1 believes that the following Stage 2 will proceed with the provision of the good if and only if all N show up. Then it is optimal for everyone to show up, and it is also ex post optimal for the meeting to follow this all-or-nothing strategy if anyone tries to test it, so long as people are very patient, that is, there is little or no discounting of payoffs in successive plays.

The second equilibrium gives everyone higher payoffs; in fact it is fully optimal. Also, it obtains when there is low discounting, that is, the waiting costs of negotiation are low, which fits naturally with our assumption of no transaction costs. Therefore, this equilibrium has some claim to attention. However, we find that it is not robust to the introduction of even small costs of attending meetings. Also, if individuals differ in their bargaining abilities, and, therefore, bear different cost shares at Stage 2, then the requirement of low discounting is even stricter than with equal shares. Therefore, we do not conclude that the repeated two-stage Coaseian procedure must yield efficient outcomes. Taking the idea of voluntary participation seriously does point out major obstacles to efficiency.

We should emphasize that our criticism pertains to the internal logic of the Coase Theorem. We do not wish to claim that public goods generally go unprovided in practice. Some large inefficiencies persist — see Olson (1996) — but groups do strive to overcome free rider problems and often succeed. Our claim is that they are unlikely to succeed if they rely solely on voluntary participation choices of individuals. Successful provision of public goods, or internalization of externalities in large groups, usually requires some form of coercion. Of course there are different types and degrees of coercion, some more palatable than others. In the concluding section we point out a particularly simple and relatively acceptable form, which is often used in practice.

2. Formal statement of the model

We now propose an extremely simple model that respects both the voluntary participation and costless enforcement assumed in the Coase Theorem. Our model is closely related to Palfrey and Rosenthal (1984) and we discuss the specific points of similarity and difference at appropriate places below. Our model takes both aspects of Coase's setup — voluntary agreement and costless enforcement — seriously. Agreements should be voluntary so that every individual has complete freedom to decide whether to enter into them, and they should be costlessly enforceable so that once an individual has made an agreement, he or she is held to all the commitments contained in the agreement. Each part of this duo has a reciprocal or negative aspect: there must be no coercion to enter any agreement ex ante, and there must be no escape from a contract ex post.

The first or voluntary agreement feature of the Coase Theorem makes a non-cooperative game formulation natural, and the second or perfectly reliable costless enforcement calls for a cooperative game formulation. Accordingly we develop a two-stage game. In the first stage, each individual decides whether to participate (choose IN) or not (choose OUT). In the second, those who have chosen IN play a cooperative game of Coaseian bargaining with costless enforcement of contracts. We first consider a single play of this game; then we allow it to be repeated.

In our model, note that the second stage is played only among those who have chosen IN at the first stage: people cannot be compelled to participate and contribute. But note also that there are no transaction costs: once you have declared yourself IN, you have no private information and no ability to engage in opportunistic behavior, and there are no obstacles to the achievement of an optimal bargain — among those who have declared IN. We will see that the last qualifying phrase carries a punch.

The equilibrium is found by backward induction. The participation decision in the first stage is made by looking ahead to the consequences of participation or non-participation and balancing the benefits against the costs.

Remember that the public good is discrete and non-excludable. The total population is N; each member gets benefit V from the good; the cost of the good is C; and M is the smallest number whose benefits cover the cost, so $MV \geq C > (M-1)V$.

We will show that as N increases and $M < N$, the likelihood that the good gets provided goes down very rapidly and a Pareto-efficient outcome is extremely unlikely. Therefore, the result contradicts the Coase Theorem in a very strong sense. We postpone further discussion of our assumptions until later, in the hope that they can better be understood in the light of the analysis and the results.

3. Single play

We begin by supposing that the two-stage game is played only once, and consider its forward-looking or subgame-perfect equilibrium. For this, we begin by finding out what happens in the second stage. That is easy: if n people have chosen IN at the first stage, where $n \geq M$, it is optimal for them to produce the good in the standard Coaseian manner. Thus, here we do not quarrel with the Coaseian argument that a negotiating meeting, once convened, reaches an outcome that is efficient for the participants. However, we do not allow the participants to coerce the non-participants. Since all participants are identical, we resolve their problem of bargaining about sharing the cost of the good by assuming equal shares. Then each of the n participants, where $n \geq M$, gets the net benefit of $V-(C/n)$, while each of the $(N-n)$ non-participants (free riders) gets V. If fewer than M people choose IN, the good is not produced. (Later we introduce further rounds that will allow reconsideration.)

We pause to discuss the relation between our model and Palfrey and Rosenthal (1984). They assume that the contribution k required from each participant is the same no matter what their numbers, whereas we fix the total cost of the good, C, so that the contribution of each participant, (C/n), is inversely proportional to their number n. They consider two cases: the ‘refund case’ where if less than M people participate each gets zero, and a 'non-refund case' where in such an eventuality each participant still pays k even though the good is not provided, so the payoffs are $-k$ each. In the non-refund case there is a 'fear' motive for an individual to choose non-participation: if too few others show up, one merely loses one's stake. This is absent from our model. In the refund case there is a 'greed' motive for non-participation: if enough others show up, then one does better by free-riding. This greed motive has less force in our model than in Palfrey–Rosenthal: if more of the others contribute, then one's own share of the cost, and, therefore, one's own saving from free riding, is smaller. Thus, in our model the rules are actually very much more favorable to generating contributions and achieving an efficient outcome than is the case in Palfrey–Rosenthal. Nonetheless, we find that the likelihood of the good being provided is generally very small. Therefore, our negative conclusion has much more force.

Now we return to our model and examine its the first stage, where individuals decide whether to participate in the meeting, looking ahead rationally to the equilibrium outcome of that stage. We begin by looking for equilibria in pure strategies. If $M \geq 2$, there is an equilibrium where everyone chooses OUT — when everyone else is choosing OUT, one person switching to IN achieves nothing. If we accept this equilibrium the Coase Theorem is already contradicted, so we proceed to look at alternatives. There are no other pure strategy equilibria that are symmetric in the sense that all players choose the same strategy. There is a whole collection of equilibria where precisely M of the N players choose IN and the rest choose OUT. But this most arbitrarily requires identical players to choose different strategies in a precisely coordinated manner and it also runs against the non-cooperative nature of the first stage. Therefore, we turn to mixed-strategy equilibria. Palfrey and Rosenthal (1984) do likewise.

The same pragmatic argument in favor of choosing the symmetric mixed-strategy equilibrium, namely the difficulty of the coordination that is required for choosing one among the asymmetric pure strategy equilibria when a subset of a large group is designated to do one thing and the rest another, is also invoked in the large literature on free-riding in acceptance of corporate takeover bids; see Bagnoli and Lipman (1988) and Holmström and Nalebuff (1992). In our context, the argument for symmetry is stronger because the coordination problem is harder: the potential participants are not even identified until they show up for the meeting, whereas in the takeover context at least the identities and addresses of all the shareholders are known in advance. Even more basic theoretical support for our choice of the symmetric mixed-strategy equilibrium comes from Crawford and Haller (1990). They consider games in which identical players have identical preferences among multiple equilibria, but must learn to coordinate over which equilibrium to play by repeatedly playing the game. They find convergence with probability 1 and in finite time to an action combination that yield all players equal stage game payoffs. In our context only the symmetric mixed-strategy equilibrium has this property.

Note that as soon as we have mixed strategies, the probability that the good will be provided in equilibrium is less than 1. Thus, our choice of equilibrium is tantamount to ruling out full efficiency. But the interest in our result comes from the fact that it is far stronger than a mere recognition of *some inefficiency*. We find that the probability of the good being provided is close to zero in most situations; that is, the outcome is close to the extreme of *total inefficiency*.

We now proceed with the calculations for Stage 1. Let P denote the probability that any one player chooses IN. Fix on one player — Herb — and consider the consequences of each of his two choices.

First suppose Herb chooses IN. If $(M-1)$ or more of the remaining $(N-1)$ people also choose IN, the good will get produced. If a total of n (where $N \geq M$) including Herb choose IN, then Herb's net benefit is $V-(C/n)$. Using the appropriate binomial probabilities of the other people's choices, Herb's expected net benefit is:

$$\sum_{n=M}^{N} \frac{(N-1)!}{(n-1)!\,((N-1)-(n-1))!} P^{n-1}(1-P)^{(N-1)-(n-1)}\left[V-\frac{C}{n}\right] \tag{1}$$

Next suppose Herb chooses OUT. If M or more of the remaining $(N-1)$ people choose IN, the good will get produced and Herb will get the free rider's benefit of V. The expected value of this is:

$$\sum_{n=M}^{N-1} \frac{(N-1)!}{n!(N-1-n)!} P^{n}(1-P)^{N-1-n}V \tag{2}$$

For a mixed-strategy equilibrium, Herb must be indifferent between the two pure choices. Equating the two expressions for his expected payoff yields an implicit equation defining the equilibrium P.

To simplify the expected payoff of IN given by (1), separate out the benefit and the cost sums, and define a new index of summation $\nu = n-1$ in the first. Then the expected payoff of IN becomes:

$$\begin{aligned}&\sum_{\nu=M-1}^{N-1} \frac{(N-1)!}{\nu!(N-1-\nu)!} P^{\nu}(1-P)^{N-1-\nu}V \\ &\quad -\sum_{n=M}^{N} \frac{(N-1)!}{(n-1)!((N-1)-(n-1))!} P^{n-1}(1-P)^{(N-1)-(n-1)}\frac{C}{n}\end{aligned} \tag{3}$$

In the first of these sums we can relabel the dummy index of summation ν and call it n instead. When we equate this expression to the benefit of OUT, namely (2), most of the terms from the first sum in (3) cancel against the sum in (2). We are left with:

$$
\begin{aligned}
0 \quad = \quad & \frac{(N-1)!}{(M-1)!((N-1)-(M-1))!} P^{M-1}(1-P)^{(N-1)-(M-1)} V \\
& - \sum_{n=M}^{N} \frac{(N-1)!}{(n-1)!((N-1)-(n-1))!} P^{n-1}(1-P)^{(N-1)-(n-1)} \frac{C}{n}
\end{aligned}
\tag{4}
$$

This equation carries the intuition that was explained before. The first term on the right-hand side is the extra benefit that Herb gets from choosing IN rather than OUT — when precisely $(M-1)$ of the other $(N-1)$ people choose IN, Herb is pivotal and can get the benefit V only by choosing IN. The other terms (the sum) on the right-hand side of (4) constitute the cost to Herb of choosing IN — when $n = M - 1$ or more of the rest choose IN, the good gets produced and Herb must pay his share of the cost. In the mixed-strategy equilibrium, Herb must be indifferent between the two pure choices. The value of P (the same for all players) adjusts in equilibrium to bring this about.

A little regrouping of factors within each term converts (4) into a much simpler form. Define:

$$b(N, M, P) = \frac{N!}{M!(N-M)!} P^{M}(1-P)^{N-M} \tag{5}$$

This is just the density of a binomial distribution, namely the probability of exactly M 'successes' in N independent Bernoulli trials when the probability of success in each trial is P. Then (4) can be written:

$$\frac{b(N, M, P)}{\sum_{n=M}^{N} b(N, n, P)} = \frac{C}{MV} \tag{6}$$

The left-hand side of (6) is the 'hazard rate' of the binomial distribution — the density at one point divided by the cumulative density to the right of this point, or the probability of exactly M successes divided by that of M or more successes. Expressed as a function of P for given N and M, the hazard rate decreases monotonically from 1 to 0 as P increases from 0 to 1. Fig. 1 shows the hazard rate for the case where $N = 6$ and $M = 2$. For larger values of N and M, unless M is almost equal to N, the decline in the hazard rate is very rapid as P increases starting at 0.

On the right-hand side we have a fraction that is less than 1, but not by too much. By the definition of $MV > C$ but $(M-1)V < C$, so:

$$\frac{M-1}{M} < \frac{C}{MV} < 1 \tag{7}$$

For small values of M, the range of $C/(MV)$ can be quite substantial, but for large M the fraction must be very close to 1.

Equilibrium can now be determined using Fig. 1. The hazard rate of exactly M IN choices is shown as the decreasing solid curve, and the cumulative probability

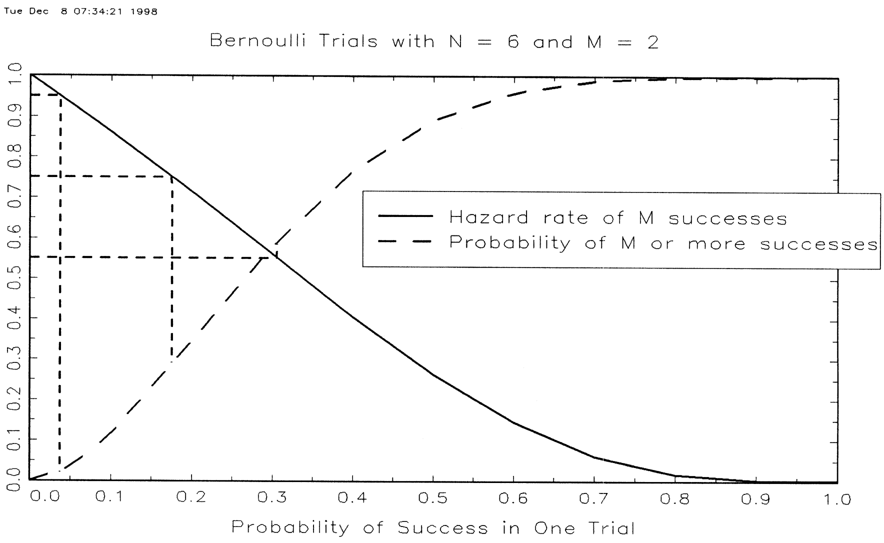

Fig. 1. Determination of equilibrium in single play.

of M or more IN choices is the increasing curve in long dashes. The magnitudes of C and V are exogenously known, and M is defined in terms of them. Then $C/(MV)$ is known, so the value of P where the hazard rate equals this ratio can be read off and the cumulative probability can be found. In the figure this is shown by means of the lines in short dashes, corresponding to the values of $C/(MV)$ that are defined below.

Now we see why the equilibrium has such anti-Coaseian properties. Since $C/(MV)$ has to be close to 1, from the hazard rate graph we see that P has to be small, and then from the cumulative probability graph we see that the probability that the good gets produced has to be small also.

The situation is least unfavorable to a Coaseian efficient outcome in two circumstances: (1) when the hazard rate does not decline very rapidly for P close to 0; and (2) if $C/(MV)$ is close to its minimum possible value of $(M-1)/M$. The first happens when M is close to N, that is, when almost everyone's participation is needed and everyone is quite likely to be pivotal. The second happens when an individual's pivotal contribution brings that individual a larger gain.

This is quite intuitive. The only incentive to contribute is that you might be the pivotal Mth person. The more by which N exceeds M, the less likely you are to be pivotal, so the less likely you are to contribute.

In addition, the larger that M is, the smaller the gain that arises from being pivotal. Suppose $V=1$, $M=2$, and that the cost C is the midpoint, 1.5, between the case where a single person would have just found it advantageous to provide the good ($C=1$) and where two would just barely have provided ($C=2$). The

Table 1
Individual probability, P, of choosing IN, and cumulative probability, Q, of success with $M = 10$

	$C = 9.1$		$C = 9.5$		$C = 9.9$	
N	P	Q	P	Q	P	Q
15	0.17	0.24×10^{-4}	0.10	0.19×10^{-6}	0.21×10^{-1}	0.60×10^{-13}
30	0.48×10^{-1}	0.76×10^{-6}	0.27×10^{-1}	0.37×10^{-8}	0.55×10^{-2}	0.66×10^{-15}
50	0.24×10^{-1}	0.31×10^{-6}	0.14×10^{-1}	0.14×10^{-8}	0.27×10^{-2}	0.23×10^{-15}
200	0.52×10^{-2}	0.14×10^{-6}	0.29×10^{-2}	0.57×10^{-9}	0.58×10^{-3}	0.86×10^{-16}

gain to the pivotal individual with $n = 2$ is $1-(1.5/2)$ or 0.25. Now suppose that $M = 1000$. Assume again that the cost is exactly at the midpoint between where $M - 1$ would barely have gained from providing, and where M would just have barely have provided, i.e. 999.5. The gain to an individual from providing the pivotal contribution, $(V - C/n)$, is now only $[1-(999.5/1000)]$ or 0.0005. The gain from being the pivotal Mth contributor is 500 times higher when $M = 2$ than when $M = 1000$.

Both of these ways in which larger numbers reduce the likelihood of efficient outcomes are of great practical pertinence. As N gets larger, it is likely to exceed M by a larger absolute number and each individual is less likely to be pivotal. Similarly, when a public good would benefit a larger number, typically M will also be larger; the dam that would protect the many that are likely to live in the flood plain of a great river is likely to cost more than a dam that would protect the few who are likely to live near a small stream. The gain to an individual from providing a pivotal contribution would then be smaller for the larger dam.

In the many important cases where M is absolutely large but substantially smaller than N, the equilibrium P, and the probability of the Coaseian outcome, will be both very small. Tables 1–3 show various combinations of parameters that demonstrate this vividly.[4]

Throughout we kept $V = 1$; this is just a normalization. Table 1 shows the results for the case where $M = 10$. We allow N to range from 15 to 200. We also consider three values of C, namely 9.1, 9.5 and 9.9. With the first of these, nine are just

Table 2
Individual probability, P, of choosing IN, and cumulative probability, Q, of success with $M = 2$

	$C = 1.1$		$C = 1.5$		$C = 1.9$	
N	P	Q	P	Q	P	Q
3	0.71	0.80	0.50	0.50	0.13	0.51×10^{-1}
6	0.31	0.59	0.18	0.28	0.37×10^{-1}	0.19×10^{-1}
15	0.11	0.51	0.59×10^{-1}	0.22	0.12×10^{-1}	0.13×10^{-1}
60	0.27×10^{-1}	0.48	0.14×10^{-1}	0.20	0.26×10^{-2}	0.11×10^{-1}

[4] The computations were carried out using the routines in Press et al. (1989, pp. 166–169).

Table 3
Individual probability, P, of choosing IN, and cumulative probability, Q, of success with $M = 50$

	$C = 49.1$		$C = 49.5$		$C = 49.9$	
N	P	Q	P	Q	P	Q
60	0.84×10^{-1}	0.60×10^{-43}	0.49×10^{-1}	0.97×10^{-55}	0.10×10^{-1}	0.11×10^{-88}
100	0.18×10^{-1}	0.27×10^{-58}	0.10×10^{-1}	0.10×10^{-70}	0.20×10^{-2}	0.26×10^{-105}
150	0.91×10^{-2}	0.74×10^{-62}	0.51×10^{-2}	0.23×10^{-74}	0.10×10^{-2}	0.48×10^{-109}
250	0.46×10^{-2}	0.56×10^{-64}	0.25×10^{-2}	0.16×10^{-76}	0.51×10^{-3}	0.28×10^{-111}

insufficient to provide the good, and with the third, 10 are just sufficient. In each case, we denote by P the probability that any one individual chooses IN, and by Q the cumulative probability that the good gets provided, that is, that M or more individuals choose IN.

Consider first the central case with $C=9.5$. In a society of 15 people, each chooses IN with a probability of about 10%. But the total probability that these choices yield 10 or more IN votes and cause the good to be produced is only 0.00000019. We can see this more easily if we think in terms of a normal approximation to the binomial. With $N = 15$ and $P = 0.1$, the number of successes regarded as a normal variate has the mean $NP = 1.5$ and the standard deviation $\sqrt{NP(1-P)} = 1.16$. Getting 10 or more successes is an event more than 7 standard deviations beyond the mean, and, therefore, exceedingly unlikely. [5]

As N increases, the probability that any one individual chooses IN decreases. We expect this, but the probability of 10 or more IN choices could still go up because there are more people making the choices. We see that this is not the case. The decrease in individual probabilities of choosing IN is the more powerful force, to the point that the probability of the good being produced very quickly falls to an even more negligible level.

The probabilities for $C = 9.1$ are a little higher, because the cost of choosing IN is lower in relation to the benefit. But the bottom line, namely the probability that the good gets produced, remains negligible. Increasing C to 9.9 makes matters worse: even with $N = 15$, the probability that any one chooses IN is now only a little over 2%, and the probability that the good gets produced is virtually zero.

Table 2 shows calculations with smaller numbers. Now $M = 2$, and the values of C are 1.1, 1.5 and 1.9. These are the values, for the case $N = 6$, that were used in Fig. 1. Now we examine the results for a whole range of N. First consider the middle column with $C = 1.5$. In a group of total size 3, each chooses IN with probability 50%, and that is also the probability that the good is produced.[6] As the

[5]Incidentally, the fact that the standard deviation is proportional to the square root of N shows why the outcome for P is not invariant to a proportional scaling up of both N and M.

[6]Incidentally, for $N = 3$ and $M = 2$, the mixed-strategy equilibrium can be found in closed form, and the probability of choosing IN is exactly 50%. This is a useful check on the accuracy of our computational procedure and of the numerical calculations and FORTRAN programs we used.

group size rises to 60, the probability that any one chooses IN falls to about 1.38%. The probability that at least two people choose IN (and, thus, the good gets produced) also falls, but levels off and asymptotes to about 20%.

Next turn to the first column, where $C = 1.1$. Here $C/(MV) = 0.55$, which is about as low a value as this ratio can possibly have. Then from Fig. 1 we see that the equilibrium P gets quite large; with $N = 6$ it is a little over 30%. The cumulative probability is almost 60%. This is about as favorable a case for the provision of the public good as one can find.

Conversely, in the third column, where C is 1.9, all the probabilities are uniformly lower, and the asymptotic probability of the good being produced in a large group is only a little over 1%.

Table 3 shows a case with numbers more appropriate to non-trivial group decision problems. Here $M = 50$, and values of N ranging from 60 to 250 are considered. The probabilities of the good being produced are uniformly close to zero.[7]

These calculations show that even when transactions costs are absent, large numbers constitute a distinct problem and can lead to grossly inefficient outcomes. Similar results obtain even when there are transaction costs; for example Mailath and Postlewaite (1990).

4. Repeated play

If the above game results in non-provision because fewer than M people show up at the meeting (choose IN at Stage 1), then everyone stands to gain from playing the whole game again. At the minimum, under our rules, there is nothing to lose: there are no transaction costs, including costs of waiting. Therefore, we now consider the effect of such repetitions. We find that they do not rescue the Coase Theorem.[8]

A fixed finite number of repetitions clearly will not help. No one has any incentive to choose IN for any but the last play, which then degenerates into the single-play model above. However, stopping the game after a finite number of repetitions is arbitrary, and not logically consistent with our assumption of zero transaction costs. Therefore, we must consider infinite repetition.

As is well known, infinitely repeated games can have multiple equilibria, each sustained by its own internally consistent expectations. This one is no exception.

[7] We hope the 0.28×10^{-111} in Table 3 is a record for the smallest number ever to appear in an economics paper.

[8] Related models of repeated attempts at coordination are Farrell and Saloner (1988) and Bolton and Farrell (1990).

We find two such equilibria (doubtless there are many more), with vastly different efficiency properties.

4.1. An inefficient equilibrium

Suppose that in any one ‘play’ or ‘round’ of the repeated game, all individuals at Stage 1 expect that the meeting at the immediately following Stage 2 will go ahead with the provision of the good if M or more people are present. Call this the ‘go-ahead’ expectation. We will show that such expectation is indeed rational — this choice is ex post optimal for the meeting — given the individuals’ responses to it. For the moment, focus on these responses. There are two effects. For each individual in each round, the incentive to choose IN is reduced because there is the prospect of further rounds if too few people show up at this round. Thus, the equilibrium P in each play is lower. But for the group as a whole, there are more rounds, and the prospect that enough people will show eventually in one of the rounds is greater for any given P. The overall effect is the balance of the two, and it turns out that the balance tilts toward even greater inefficiency.[9]

We illustrate this by considering the special case where $N=2$ and $M=1$, and then build the general case. We also develop the theory with discounting, and then take the limit as the discount rate goes to zero, as required for our assumption of zero transaction costs. Write β for the discount factor. As usual, $\beta = 1/(1+r)$ where r is the discount rate, so the no-transaction-cost limit corresponds to $r=0$ or $\beta=1$. Also, write W for the equilibrium payoff or value of the game to each player.

Now the payoff matrix for the two players at Stage 1 is as shown in Table 4. The notation and the explanation is as follows. If both players choose IN, the good is provided and the cost shared; each gets $V-\frac{1}{2}C$. If one chooses IN and the other OUT, then given the temporarily assumed expectations about Stage 2, the good is still provided, the IN player bears the cost and gets $V-C$; the OUT player free-rides and gets V. If both choose OUT, the good is not provided and the game

Table 4
Payoff matrix for Stage 1 under go-ahead expectations

		Player B	
		IN	OUT
Player A	IN	$V-\frac{1}{2}C, V-\frac{1}{2}C$	$V-C, V$
	OUT	$V, V-C$	$\beta W, \beta W$

[9] It might be argued that if fewer than M people turn up in the first round, they can commit themselves not to go ahead unless everyone turns up. But they cannot make a similar commitment on behalf of others who have not shown up at this round and might do so at later rounds.

is repeated starting next period; the payoff from this is W to each player starting one period later, so its discounted present value today is βW.

In the mixed-strategy equilibrium each player should be indifferent between choosing IN and OUT, and the common payoff of the two is by definition the value of the game, W. Therefore:

$$W = P\left(V - \frac{1}{2}C\right) + (1 - P)(V - C) \tag{8}$$

and

$$W = PV + (1 - P)\beta W, \quad \text{or } W = PV/[1 - \beta + \beta P] \tag{9}$$

These two simultaneous equations in P and W as unknowns can be solved for the equilibrium.

Fig. 2 graphs these two equations, and shows that there is a unique solution in the range [0, 1] for P. The values are normalized so that $V = 1$, and for sake of illustration I have chosen $C = 0.6$ so that $V - C = 0.4$ and $V - \frac{1}{2}C = 0.7$. (There is another solution with $P > 1$ but that is economically irrelevant.) At this solution the value to each player is more than $(V - C)$ — what he would get by paying the full cost of the good himself — but is less than $(V - \frac{1}{2}C)$ — the efficient outcome where the cost is shared. Thus, the total payoff to the pair is less than $(2V - C)$: some inefficiency remains.

Given this solution for Stage 1, consider the ex post optimality of actions for the IN group at Stage 2. If both happen to have chosen IN, the group of 2 should

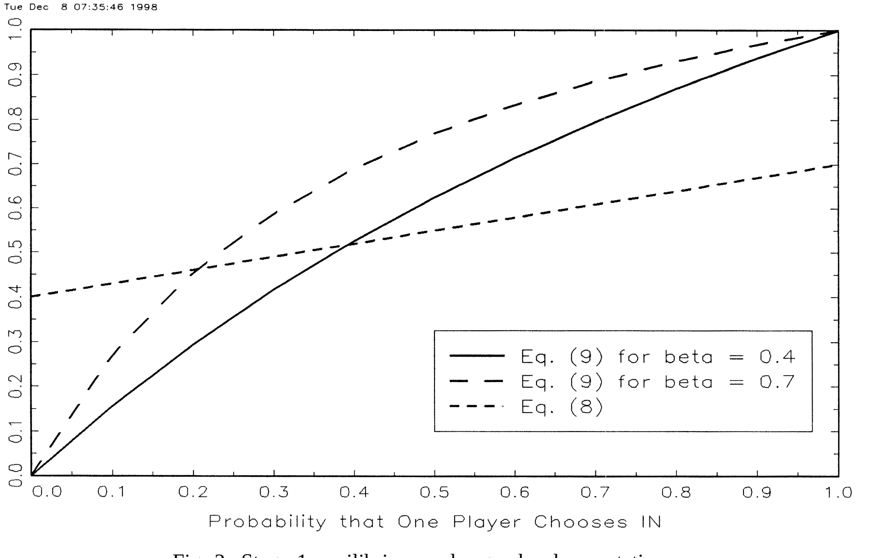

Fig. 2. Stage 1 equilibrium under go-ahead expectations.

clearly go ahead. If, say, A has chosen OUT, then B gets $V - C$ by going ahead, and βW by deviating and sending the game to another round. (By Raiffa's Theorem it is enough to check single deviations.) This deviation is unprofitable if $V - C > \beta W$. We now check this.

Eqs. (8) and (9) can be written as:

$$V - W = C\left(1 - \frac{1}{2}P\right), \quad V - W = (1 - P)(V - \beta W)$$

Therefore:

$$(V - \beta W)/C = \left(1 - \frac{1}{2}P\right) \Big/ (1 - P) > 1$$

so $V - C > \beta W$ as required. Thus, we have a self-consistent equilibrium of expectations and actions.

Now we examine the properties of this equilibrium. Fig. 2 shows that the value W lies between $V - C$ and $V - \frac{1}{2}C$. Thus, each player gets higher payoff than by bearing the whole cost of the good, but less than that in the efficient outcome. Thus, some inefficiency remains.

As β increases, the curve representing (9) shifts up as shown in the figure; the equilibrium shifts to the left along the line (8) and the expected value of the game to each player falls. Thus, greater patience — lower transaction cost — paradoxically leaves both players worse off. The reason is that greater patience makes each shirk more in any one round (P goes down). In the limit with total patience, $\beta \to 1$; then $P \to 0$ and $W \to V - C$, and the outcome is as if each player had to bear the full cost of the good.

The result for the general case is similar. The payoffs from choosing IN is:

$$W = \sum_{n=1}^{M-1} b(N-1, n-1, P)\beta W + \sum_{n=M}^{N} b(N-1, n-1, p)(V - C/n) \tag{10}$$

and that from choosing OUT is:

$$W = \sum_{n=1}^{M} b(N-1, n-1, P)\beta W + \sum_{n=M+1}^{N} b(N-1, n-1, P)V \tag{11}$$

As $\beta \to 1$, (11) becomes:

$$W\left[1 - \sum_{n=1}^{M-1} b(N-1, n-1, P)\right] = V \sum_{n=M}^{N} b(N-1, n-1, p)$$

The two probability sums are equal, so either each of the sums is zero (which corresponds to $P \to 0$), or $W = V$. But the latter is impossible since (10) shows W to be a weighted average of terms all of which are less than V. (This corresponds to the irrelevant solution $P > 1$ of the two-player case.)

Now write (10) as:

$$W = \frac{\sum_{n=M}^{N} b(N-1, n-1, p)(V - C/n)}{1 - \beta + \beta \sum_{n=1}^{M-1} b(N-1, n-1, P)}$$

As $\beta \to 1$ and $P \to 0$, the numerator and the denominator both go to zero. But the first term in the numerator combines with the probability sum in the denominator to form the hazard rate for M successes in N trials, which goes to 1. Then $W \to V - C/M$. The value to each player is the same as if only the minimal coalition of M people forms; there is no benefit of lower cost from larger coalitions of participants. The reason is again the increase in the incentive to shirk in any one play.

The total value to all N players is:

$$NW = NV - (N/M)C < NV - C$$

so the outcome is inefficient. The measure of this inefficiency, or the shortfall in the total payoff, is $(N/M)C - C$. For large numbers, where as we saw above, C/M is approximately equal to V, the shortfall is almost $NV - C$. That is, the inefficiency is almost 100%.

4.2. An efficient equilibrium

Now suppose the expectation about Stage 2 of any round is as follows. If all N players have chosen IN, the meeting will go ahead and produce the good. If not, the meeting will adjourn without providing for the good, and the game will proceed to its next round. Call this the 'all-or-nothing' expectation. Again we will show later that this choice is rational for the meeting, given the individuals' responses in its expectation, and provided the discount factor is sufficiently close to 1. For now we examine the individual responses at Stage 1.

Given these expectations, the Stage 1 payoff matrix changes to that shown in Table 5.

If $V - \frac{1}{2}C > \beta W$, then IN is the dominant strategy for each player, and the resulting payoff is $W = V - \frac{1}{2}C$, confirming the requirement for dominance. It only remains to check that, given the resulting Stage 1 equilibrium, the all-or-nothing response is optimal at Stage 2.

If both players have chosen IN at Stage 1, going ahead is clearly optimal. But if,

Table 5
Payoff matrix for Stage 1 under all-or-nothing expectations

		Player B	
		IN	OUT
Player A	IN	$V - \frac{1}{2}C, V - \frac{1}{2}C$	βW, βW
	OUT	βW, βW	βW, βW

say, A tests the matter by staying OUT at Stage 1, what should B do? A deviation by going ahead and providing the good gets him $V - C$. Adherence to the strategy gets $\beta W = \beta(V - \frac{1}{2}C)$. Therefore, the deviation is unprofitable if:

$$\beta > (V - C)/\left(V - \frac{1}{2}C\right)$$

This is true for β sufficiently close to 1. Therefore, with sufficiently patient players (sufficiently low transaction cost of bargaining), the all-or-nothing response is credible and the resulting (efficient) equilibrium is subgame-perfect.

The analysis for the general case is similar: the all-or-nothing strategy is ex post optimal if:

$$\beta > \frac{V - [C/(N-1)]}{V - (C/N)} \tag{12}$$

This is again true for β sufficiently close to 1.

Note the difference between this and the claims of some Coaseians: they argue that someone can take the lead *before* the meeting (at Stage 1) and threaten a would-be free rider that unless everyone participated the good would not be provided. But no assertions before the meeting carry any automatic credibility about what the meeting will decide. The argument here is that *during* the meeting at Stage 2, the all-or-nothing choice will be found ex post optimal. Therefore, this is a credible expectation to entertain for someone doing the calculation of IN vs. OUT in isolation at Stage 1. In other words, we have found another self-consistent equilibrium of the repeated game.

4.3. Choosing between the equilibria

When a game has multiple equilibria, one must look for some other consideration that will help select one of them. There are different criteria of this kind, and we examine some of them.

4.3.1. Focal point

Schelling (1960, Chap. 3) introduced the best-known consideration of this kind, namely a focal point. If the expectations of all players can converge on one of the equilibria, then everyone will play his part in it, expecting everyone else to do likewise. Schelling showed how various historical, cultural or linguistic forces can create such a convergence of expectations on one equilibrium. We have not specified any such details, but there is an argument which suggests that the efficient equilibrium has a greater claim to be a focal point.

This comes from the comparison of the payoffs themselves. Given our assumption of zero transaction costs, every player in isolation at Stage 1 has full knowledge of the data (N, V and C), and can costlessly figure out both or all equilibria, each with its self-consistent expectations. Then each thinks whether everyone else will think that . . . one of these is to be favored. He knows that everyone else has done the same calculation, and so everyone knows that one of the equilibria offers higher payoffs to everyone than any other. Therefore, each should think that all will think that . . . it should be chosen.

4.3.2. Robustness to small transaction costs

Many would regard the focal point argument to be compelling, and we admit its force. But we point out another consideration which cuts the other way. This is to ask that the equilibrium be robust to a small perturbation of the model. Our main focus is on the case of zero transaction costs. But if we were to change the model by introducing very small transaction costs, would it have an equilibrium close to the one for the zero-transaction-cost case?[10]

We have already constructed the model by introducing one form of transaction cost, namely the cost of waiting while the negotiation goes on through its rounds. This is implicit in the discount factor β. Although our main interest was in the limiting case as $\beta \rightarrow 1$, in the process we found that the limit proceeded continuously. This was true of the inefficient equilibrium as well as the efficient one; both are robust to the introduction of a small cost of waiting.

Now we consider another kind of cost, namely of attending a meeting. Here we find a great difference: the inefficient equilibrium above is robust, while the efficient one is not — it disappears when there is some cost of each type, waiting and attending meeting, no matter how small each may be. Again we show this in the two-person context; the general case is then obvious.

Consider the inefficient equilibrium first. Let ϵ denote the cost of attending a meeting, measured in the same units as V and C. Then the payoff matrix of Stage 1, given the go-ahead expectations about Stage 2, is shown in Table 6. The equilibrium is again in mixed strategies, and the equations, replacing (8) and (9), are:

$$W = P\left(V - \frac{1}{2}C - \epsilon\right) + (1 - P)(V - C - \epsilon) \tag{13}$$

and

Table 6
Payoff matrix for Stage 1 under go-ahead expectations and attendance cost

		Player B	
		IN	OUT
Player A	IN	$V - \frac{1}{2}C - \epsilon, V - \frac{1}{2}C - \epsilon$	$V - C - \epsilon, V$
	OUT	$V, V - C - \epsilon$	$\beta W,\ \beta W$

[10] Anderlini and Felli (1997) introduce small contracting costs, and consider a different implication, namely a hold-up problem. This also leads to an anti-Coase result.

$$W = PV + (1 - P)\beta W, \quad \text{or } W = PV/[1 - \beta + \beta P] \tag{14}$$

The solution can be found as in Fig. 2. The sole difference is that the straight line representing (8) is vertically lowered by ϵ to get (13). This reduces P and V, and the change is small when ϵ is small.

The condition for the go-ahead behavior to be ex post optimal at Stage 2 is $V - C > \beta W$, and it is easy to verify that it is satisfied. In fact it is easier to satisfy now, since the cost of attending another meeting makes it even more desirable (albeit only slightly so when the cost is small) for one person to go ahead without waiting for the other to show up in the next round.

Thus, we continue to have the inefficient equilibrium with self-sustaining expectations, and it changes continuously as a small cost of attendance is introduced.

The efficient equilibrium changes dramatically. The Stage 1 payoff matrix, given all-or-nothing expectations about Stage 2, becomes as shown in Table 7. Choosing IN is no longer the dominant strategy, and an equilibrium in mixed strategies must be found.

Using the by-now familiar notation and technique, the two equations defining the probability P of playing IN and the value W of the game are:

$$W = P\left(V - \frac{1}{2}C - \epsilon\right) + (1 - P)(\beta W - \epsilon) \tag{15}$$

and

$$W = P\beta W + (1 - P)\beta W = \beta W \tag{16}$$

So long as $\beta < 1$, no matter how small the difference (no matter how small the cost of waiting), Eq. (16) admits only one solution, namely $W = 0$. Then Eq. (15) gives:

$$P = \epsilon/\left(V - \frac{1}{2}C\right)$$

Thus, the probability of participation is very small, and the value of the game is very low, namely zero. The only good thing about this situation is that it cannot be an equilibrium with self-sustaining expectations. If one person tests it out by

Table 7
Payoff matrix for Stage 1 under all-or-nothing expectations and attendance cost

		Player B	
		IN	OUT
Player A	IN	$V - \frac{1}{2}C - \epsilon, V - \frac{1}{2}C - \epsilon$	$\beta W - \epsilon,\ \beta W$
	OUT	$\beta W,\ \beta W - \epsilon$	$\beta W,\ \beta W$

staying OUT, at Stage 2 the other would find it optimal to wait and go another round if $\beta W > V - C$, which is not true as $W = 0$ but $V > C$.

Thus, the efficient equilibrium does not survive the simultaneous existence of even extremely small costs of waiting and of attending meetings. The attendance cost brings a 'no-refund' feature to the game, analogous to the case considered by Palfrey and Rosenthal (1984), and, therefore, further increases the temptation to free-ride. But in our setting of the repeated game, this makes a much more dramatic difference.

4.3.3. Unequal cost shares

Our assumption throughout the formal analysis has been that whenever n ($\geq M$) individuals find themselves IN at Stage 2 and provide the good, they share its cost equally. This was harmless in the cases where the equilibrium had very low participation probabilities and was, therefore, very inefficient. But the efficient equilibrium of the repeated game is much more vulnerable to relaxation of this assumption. Suppose $(N - 1)$ people find themselves present at Stage 2, while the remaining person is testing out their resolve to go through with the strategy of letting the game go another round and wait for everyone to show up. For this strategy to be optimal, the players must now be sufficiently patient, or their discounting of the future must be sufficiently low, to make it optimal for them to outwait even the Nth person who is most reluctant to join, that is, the one who faces the prospect of the highest cost share among all individuals. Then condition on β that replaces (12) is even stricter, that is, β must be even closer to 1 than before to sustain the efficient equilibrium. This is not strictly a problem in the simplest Coaseian world with zero transaction costs, where the discount factor can be assumed to be as close to 1 as needed. But it does make the efficient equilibrium even less robust to small changes in the assumptions, and, therefore, even more suspect as a practical guide.

5. Conclusions

What can one conclude from this? Some people would accept the argument that when one equilibrium is Pareto-better than another, it should emerge as a focal point. They would, therefore, say that we have pointed out a new non-cooperative way of achieving Coaseian efficiency. Others would regard the non-robustness to the introduction of very small transaction costs as fatal; they would say that we have disproved the Coase Theorem. We prefer not to take a dogmatic stand on the issue, but we do say that at the very least we have raised some serious doubts about the validity of Coaseian claims that in the absence of transaction costs, starting from a total tabula rasa, a set of individuals will create procedures that lead to Pareto-efficient outcomes.

When we cast doubt on the claims of universal efficiency on the basis of the

Coase Theorem, and exhibit equilibria that are grossly inefficient, our quarrel is with the logic of that argument. We do not wish to push this so far as to claim that public goods will almost never get provided in large societies. Even if a group starts trapped in such a cycle, where few or none participate, the manifest inefficiency will prompt some efforts at remedies. Providing such remedies is itself a public good and, therefore, subject to similar difficulties, but these are often overcome in practice. To examine how, and, therefore, to draw policy prescriptions, would take us too long in what is already a long paper. Therefore, we leave that task for another occasion. But in conclusion we wish to point out one possible, and commonly used, approach.

The starting point is the observation that every groups faces several collective action problems simultaneously. Individuals do not have the option of participation (in our jargon the choice of IN and OUT) separately for each issue. The choice must be made once and for all; choosing IN reveals one's identity and subjects one to compulsory participation in all the public issues facing that group. We see this as deliberate strategy, the idea being that every individual will derive private benefit from participation in some issue, and this will be sufficient to induce him to choose IN for the whole package. For example, getting a driving license makes one liable for jury service. Such bundling, and restricting individuals to say IN or OUT to the whole bundle, is a kind of coercion. But it is a relatively gentle kind of coercion, relying on individuals' self-interest, and, therefore, not unlike the price system in its rationing role.

This is how non-governmental or special interest groups often overcome free rider problems; labor unions, the AARP, etc. offer enough excludable private benefits to induce people to become members, and then a part of their membership fee is used in lobbying for the non-excludable public (really, group) good. The role of such 'selective incentives' in solving the collective action problems of interest groups was examined by Olson (1965, p. 51) and Wilson (1974, pp. 33–34). Governments can be regarded as similar mechanisms writ large.

If this method is important for achieving voluntary participation in public good provision in practice, it also offers a new explanation of why several private excludable goods are publicly provided. Economists are often puzzled by this phenomenon, and almost unanimous in advocating privatization of these activities. But the public provision of private goods may be playing an important role: packaging them with other genuinely public goods may serve to give individuals sufficient incentives to participate in the provision of those latter goods, which might otherwise suffer because of free riding.[11] In other words, the bundling with private goods can induce selfish individuals to sign on to 'society' and to participate in its public good activities.

[11] Of course for this to work, competing private supply of these private goods will have to be forbidden.

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